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BRIEF COMMUNICATION

DEPENDENCE OF SETTLING VELOCITY ON PARTICLE CONCENTRATION IN A FLUIDIZED BED OF SPHERICAL PARTICLES

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INTRODUCTION

The early work of Richardson & Zaki (1954) indicated that the relation between particle concentration and the settling velocity of suspended spherical particles can be approximated with good accuracy by a relation of the form

$$v_p = v_0(1-\alpha)^{n-1}. \qquad [1]$$

Here v_p is the mean particle velocity relative to the fluid, v_0 is the single-particle terminal velocity and α is the volume fraction of particles. The results of Richardson & Zaki (1954) suggest that the exponent n is primarily a function of the terminal Reynolds number for a single particle, $\mathrm{Re} = \rho_f v_0 d_p/\mu_f$. However, they also found that n was a weak function of the ratio of particle-to-channel diameter, d_p/D, when this ratio is significantly greater than zero ($> 10^{-2}$). They presented correlations for n as a function of Re and d_p/D which are best-fits to their experimental data.

For the limiting situation where $d_p/D \to 0$, Richardson & Zaki (1954) and Wen & Yu (1966) analytically predicted the variation of n with Re by considering the force balance on the particles in specific flow regimes. The variation of n with Re was similarly predicted by Wallis (1969) who combined the results of a force-balance analysis with experimental information.

As noted by Wallis (1969), [1], together with an appropriate correlation for n, can be used very effectively in the 1-D analysis of fluidized beds and certain other two-phase flow circumstances. The usefulness of these relations in analyses of this type has provided the incentive to develop correlations for n which are as accurate as possible.

The object of the present communication is to point out an alternative means of predicting the dependence of n on Re which is independent of direct measurements in fluidized systems. This approach takes advantage of the fact that as the concentration of particles approaches the value for incipient fluidization, the value of n must be such that the system flow and pressure-drop behavior approaches that of a rigid bed of packed spheres.

DISCUSSION

The friction pressure drop associated with a packed bed of spheres is assumed here to be given by the relation of Ergun (1952), which includes viscous and inertia effects:

$$-\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)_f = 150\frac{\alpha^2}{(1-\alpha)^3}\frac{\mu_f u_s}{d_p^2} + 1.75\frac{\alpha}{(1-\alpha)^3}\frac{\rho_f u_s^2}{d_p}. \qquad [2]$$

Here, u_s is the superficial fluid velocity and d_p is the particle diameter. At incipient fluidization, the friction pressure drop in the bed must just balance the weight of the particles, i.e.

$$-\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)_f = \alpha(\rho_p - \rho_f)g, \qquad [3]$$

where ρ_p and ρ_f are the particle and fluid densities. It can easily be shown that

$$g(\rho_p - \rho_f)d_p = \frac{3\rho_f v_0^2 C_D}{4}, \qquad [4]$$

where C_D is the drag coefficient for a single particle at its terminal velocity. Noting that the actual fluid velocity through the packed bed, v_f, is given by

$$v_f = \frac{u_s}{(1-\alpha)}, \qquad [5]$$

[1]–[5] can be combined to obtain

$$v_f = \frac{v_0(1-\alpha)^2 \,\mathrm{Re}\, C_D}{200\,\alpha}\left(\frac{\sqrt{1+4\Omega}-1}{2\Omega}\right), \qquad [6]$$

where

$$\Omega = 5.83 \times 10^{-5}\frac{(1-\alpha)^3}{\alpha^2}\mathrm{Re}^2 C_D. \qquad [7]$$

For a fluidized system, v_f must equal v_p, and hence at incipient fluidization, [1] and [6] must be equal. Data presented in Wallis (1969) and other previous studies suggest that incipient fluidization for beds of spherical particles corresponds approximately to $\alpha = 0.6$. Assuming that $\alpha = 0.6$ and setting [1] equal to [6] yields the following relation for n:

$$n = 1.09\left\{7.54 - \ln\left[\frac{\sqrt{1+4.15\times 10^{-5}\,\mathrm{Re}^2 C_D}-1}{2.07\times 10^{-5}\,\mathrm{Re}}\right]\right\}. \qquad [8]$$

Using the well-known Schiller & Nauman (1933) relation for the drag coefficient for Re < 1000 and $C_D = 0.44$ for Re $\geqslant$ 1000:

$$C_D = \frac{24(1+0.15\,\mathrm{Re}^{0.687})}{\mathrm{Re}} \quad \text{for} \quad \mathrm{Re} < 1000 \qquad [9a]$$

and

$$C_D = 0.44 \quad \text{for} \quad \mathrm{Re} \geqslant 1000; \qquad [9b]$$

[8] is a complete prediction of n as a function of Re. The resulting variation is plotted in figure 1. It is interesting to note that with [9a,b], [8] predicts that as $\mathrm{Re} \to 0$, $n \to 4.74$, and as $\mathrm{Re} \to \infty$, $n \to 2.41$. The analysis of Wen & Yu (1966) indicates values of $n = 4.65$ and $n = 2.35$, respectively, for these limits. The experimentally determined values of n from the data of Richardson & Zaki (1954) are also shown in figure 1. The prediction of [8] is seen to agree reasonably well with the data.

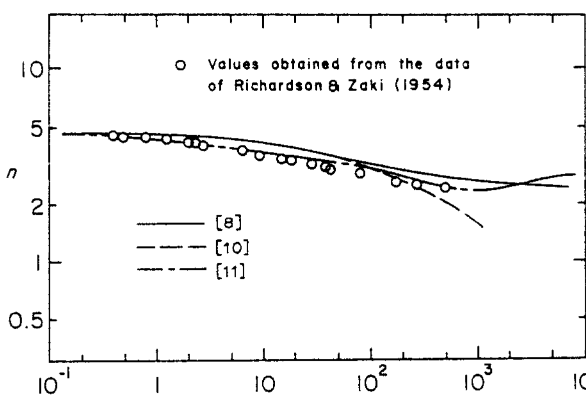

Figure 1. Comparison of predicted variations of n with that determined from the measured data of Richardson & Zaki (1954).

As noted above, [8] is derived assuming that [1] and [6] must be equal at $\alpha = 0.6$. It is easily shown that although changing the choice of α alters the coefficients in [8], the predicted variation of n with Re is not very sensitive to the choice of α. Changing α by ± 0.05 produces a $<3\%$ change in the predicted n value at low Re and a $<5\%$ change in n at high Re.

It is also worth noting that if the second term on the r.h.s. of [2] is neglected, which neglects inertia effects, [8] becomes

$$n = 1.09\,[7.54 - \ln(C_D \mathrm{Re})]. \qquad [10]$$

If [10] is used with [9a,b] to predict n, the resulting variation deviates from that for [8] only at high Re, as shown by the dotted curve in figure 1. This implies that the leveling-off of n to a constant value as $\mathrm{Re} \to \infty$ is due to the combined effects of constant C_D and the increasing importance of inertia effects on the drag characteristics of the system.

Also shown in figure 1 is the curve predicted by the relation suggested by Wallis (1969):

$$n = 4.7\left(\frac{1 + 0.15\,\mathrm{Re}^{0.687}}{1 + 0.253\,\mathrm{Re}^{0.687}}\right). \qquad [11]$$

This relation agrees better with the data than [8]. However, this equation was derived using theoretical arguments together with information on the trends in the results of Richardson & Zaki (1954). In this sense, it is somewhat fitted to these results.

It is interesting to note, however, that although the curve representing [11] fits the data well, it exhibits a complex behavior at high Re, dropping slightly near $\mathrm{Re} = 1000$ before rising and leveling-off at a limiting value of $n = 2.79$ as $\mathrm{Re} \to \infty$. In contrast, [8], derived in the present analysis, suggests that n monotonically decreases toward a limiting value of $n = 2.41$ as $\mathrm{Re} \to \infty$, which is more consistent with the observed trend in the data shown in figure 1. In addition, the limiting value of n as $\mathrm{Re} \to \infty$ for [11] is somewhat higher than that suggested by the results of Richardson & Zaki (1954) and Wen & Yu (1966).

It has been shown here that by combining relations for the known behaviors of fluidized and packed beds of spheres at the condition of incipient fluidization, a new relation for $n(\mathrm{Re})$ is obtained which agrees well with experimental data over a wide range of Reynolds number. In addition, the analysis presented here provides insight into the role of inertia effects on the variation of n, and demonstrates the necessary link between characteristics of fluidized and packed beds near the condition of incipient fluidization.

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